TRIANGLES

7
CHAPTER

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- Congruent triangles
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ANGLE SUM PROPERTY FOR TRIANGLE

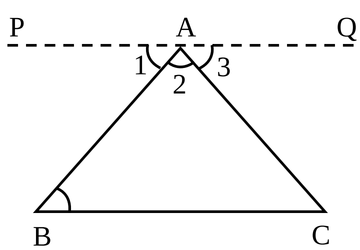
Theorem :

Prove that sum of all three angles is 180° or 2 right angles.

Given : ΔABC

To prove : $\angle A + \angle B + \angle C = 180°$

Construction : Draw PQ || BC, passes through point A.



Proof : $\left.\begin{array}{l} \angle 1 = \angle B \\ \text{and } \angle 3 = \angle C \end{array}\right\}$ alternate angles $\because PQ \parallel BC$ ....(i)

$\because$ PAQ is a line

$\therefore \angle 1 + \angle 2 + \angle 3 = 180°$ (linear pair application)

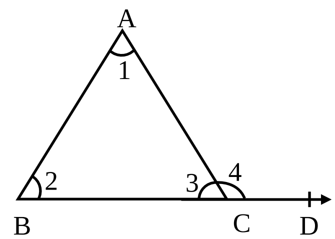
$\angle B + \angle 2 + \angle C = 180°$

$\angle B + \angle CAB + \angle C = 180°$

= 2 right angles.

Proved.

Theorem : If one side of a triangle is produced then the exterior angle so formed is equal to the sum of two interior opposite angles.



Means $\angle 4 = \angle 1 + \angle 2$

Proof : $\angle 3 = 180° - (\angle 1 + \angle 2)$ ….(1)

(by angle sum property)

and BCD is a line

$\therefore \angle 3 + \angle 4 = 180°$ (linear pair)

or $\angle 3 = 180° - \angle 4$ …..(2)

by (1) & (2)

$180° - (\angle 1 + \angle 2) = 180° - \angle 4$

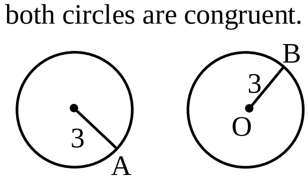
$\Rightarrow \angle 1 + \angle 2 = \angle 4$ Proved.

CONGRUENT FIGURES

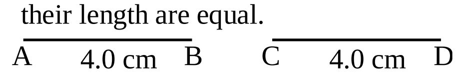
Two geometrical figures having exactly the same shape & size are known as congruent figures. Lines, polygons, circles etc. can congruent.

Note :

(1) If radius of a circle is same as other circle then only both circles are congruent.



(2) Two line segment are congruent only when their length are equal.



CONGRUENT TRIANGLES

Two triangles are congruent if and only if one of them can be made to superpose on the other, so as to cover it exactly.

Thus, congruent triangles are exactly identical.

Example 1 : If $\Delta ABC \cong \Delta DEF$ then we have :

$\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$; and

AB = DE, BC = EF and AC = DE.

Example 2 If $\Delta ABC \cong \Delta EDF$ then we have:

$\angle A = \angle E$, $\angle B = \angle D$, $\angle C = \angle F$; and

AB = ED, BC = DF and AC = EF.

Note :

(1) Every triangle is congruent to itself, i.e., $\Delta ABC \cong \Delta ABC$.

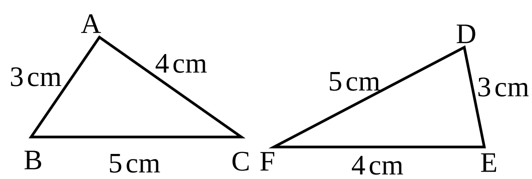
(2) If $\Delta ABC \cong \Delta DEF$ then $\Delta DEF \cong \Delta ABC$.

(3) If $\Delta ABC \cong \Delta DEF$, and $\Delta DEF \cong \Delta PQR$, then $\Delta ABC \cong \Delta PQR$.

(4) 'c.p.c.t.' for 'corresponding parts of congruent triangles'.

➢ CRITERIA FOR CONGRUENT TRIANGLES

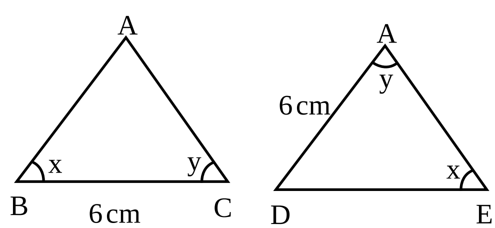
(1) SSS (Side Side Side)



$\therefore$ By SSS criteria $\Delta ABC \cong \Delta EDF$

$\therefore$ $\angle A = \angle E$, $\angle B = \angle D$, $\angle C = \angle F$ (c.p.c.t.)

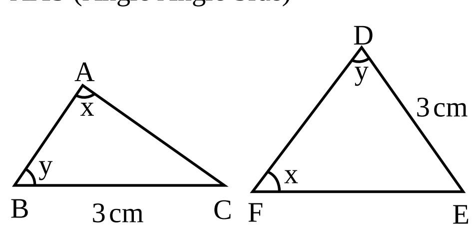
(2) ASA (Angle Side Angle)



$\therefore$ By ASA criteria $\Delta ABC \cong \Delta DEF$

$\therefore$ $\angle A = \angle D$, AB = DE, AC = DF (c.p.c.t.)

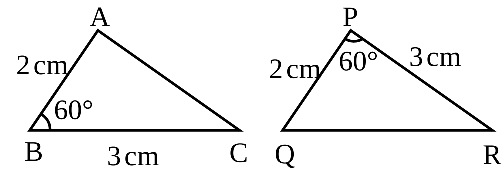
(3) AAS (Angle Angle Side)



$\therefore$ By AAS, $\Delta ABC \cong \Delta FDE$

$\therefore$ $\angle C = \angle E$, AB = FD, AC = FE (c.p.c.t.)

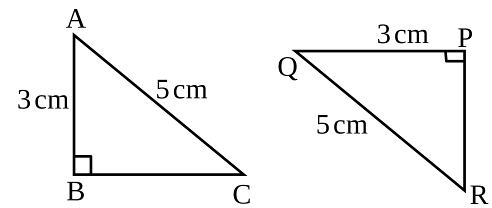
(4) SAS (Side Angle Side)



By SAS, $\Delta ABC \cong \Delta QPR$

$\therefore$ $\angle A = \angle Q$, $\angle C = \angle R$, AC = QR (c.p.c.t.)

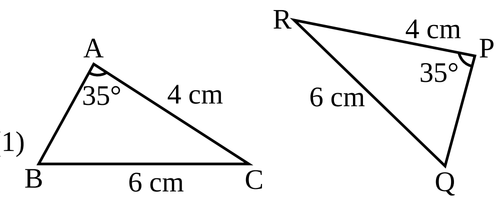
(5) RHS (Right Hypotenuse Side)



$\therefore$ By RHS, $\Delta ABC \cong \Delta QPR$

$\therefore$ $\angle A = \angle Q$, $\angle C = \angle R$, BC = PR (c.p.c.t.)

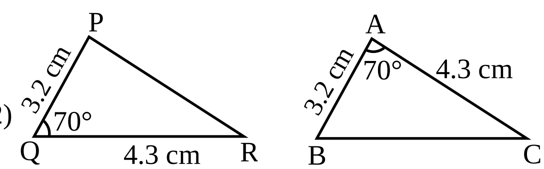
Eg.:(1)



Not congruent

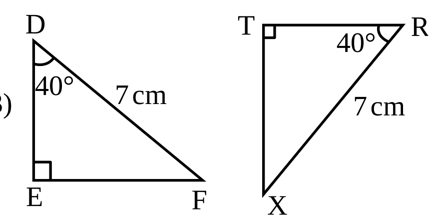
($\because$ SSA is not a rule)

(2)



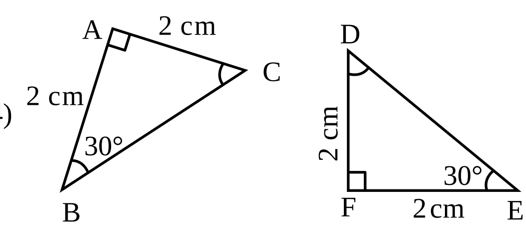
SAS

(3)



AAS (not RHS)

(4)



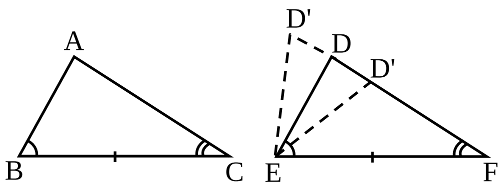
SAS (not RHS)

Theorem 1 : If two angles and the included side of one triangle are equal to two angles and the included side of other triangle, then both triangles are congruent.

Proof :

Given : ΔABC and ΔDEF in which

$\angle ABC = \angle DEF$, $\angle ACB = \angle DFE$ and $BC = EF$.



To prove : $\Delta ABC \cong \Delta DEF$.

Proof :

Case I

Let $AC = DF$.

In this case, $AC = DF$, $BC = EF$ and $\angle C = \angle F$.

$\therefore$ $\Delta ABC \cong \Delta DEF$ (SAS-criteria)

Case II

If possible, let $AC \neq DF$.

Then, construct $D'F = AC$. Join $D'E$.

Now, in ΔABC and $\Delta D'EF$, we have $AC = D'F$, $BC = EF$ and $\angle C = \angle F$.

$\therefore$ $\Delta ABC \cong \Delta D'EF$ (SAS-criteria)

$\therefore$ $\angle ABC = \angle D'EF$ (c.p.c.t)

But, $\angle ABC = \angle DEF$ (given)

$\therefore$ $\angle D'EF = \angle DEF$.

This is possible only when D and D' coincide.

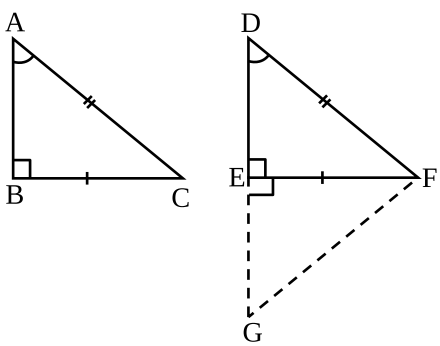
$\therefore$ $\Delta ABC \cong \Delta DEF$.

Theorem 2 : Two right-angled triangles are congruent if one side and the hypotenuse of the one are respectively equal to the corresponding side and the hypotenuse of the other. (i.e. RHS)

Given : Two right-angled triangles ΔABC & ΔDEF in which $\angle B = \angle E = 90°$, $BC = EF$ and $AC = DF$.

To prove : $\Delta ABC \cong \Delta DEF$.

Construction : Produce DE to G such that $GE = AB$. Join GF.



Proof : In ΔABC and ΔGEF, we have :

$AB = GE$ (construction),

$BC = EF$ (given), $\angle B = \angle FEG = 90°$

$\therefore$ $\Delta ABC \cong \Delta GEF$ (SAS-criteria)

$\therefore$ $\angle A = \angle G$ and $AC = GF$ (c.p.c.t.)

Now, $AC = GF$ and $AC = DF \Rightarrow GF = DF$

$\Rightarrow \angle G = \angle D \Rightarrow \angle A = \angle D$ $[\because \angle G = \angle A]$

Now, $\angle A = \angle D$, $\angle B = \angle E \Rightarrow 3^{rd}\ \angle C = 3^{rd}\ \angle F$.

Thus, in ΔABC and ΔDEF, we have:

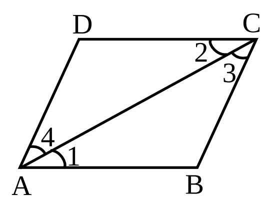
$BC = EF$, $AC = DF$ and $\angle C = \angle F$.

$\therefore$ $\Delta ABC \cong \Delta DEF$ (SAS-criteria).

❖ EXAMPLES ❖

Ex.1 Prove that diagonal of a parallelogram divides it into two congruent triangles.

Sol. Let ABCD is a parallelogram and AC is a diagonal.



(By SSS) : In ΔABC and ΔADC

$AB = CD$ (opp. sides of $||^{gm}$)

$BC = AD$ (opp. sides of $||^{gm}$)

$AC = AC$ (common)

$\therefore$ By SSS, $\Delta ABC \cong \Delta CDA$ proved

{other results : $\angle 1 = \angle 2$, $\angle 3 = \angle 4$, $\angle B = \angle D$ (c.p.c.t.)}

(By ASA) : In ΔABC and ΔADC

$\angle 1 = \angle 2$ (alternate)

$AC = AC$ (common)

$\angle 3 = \angle 4$ (alternate)

$\therefore$ By ASA, $\Delta ABC \cong \Delta CDA$

{other results : $\angle B = \angle D$, $AB = CD$, $BC = AD$ (c.p.c.t.)}

(By AAS) : In ΔABC and ΔADC

$\angle 1 = \angle 2$ (alternate)

$\angle 3 = \angle 4$ (alternate)

$BC = AD$ (opp. sides)

$\therefore$ $\Delta ABC \cong \Delta CDA$

{other results : $AB = CD$, $\angle B = \angle D$, $AC = AC$ (c.p.c.t.)}

(By SAS) : In ΔABC and ΔADC

AB = CD (opp. sides of $\|^{gm}$)

$\angle 1 = \angle 2$ (alternate)

AC = AC (common)

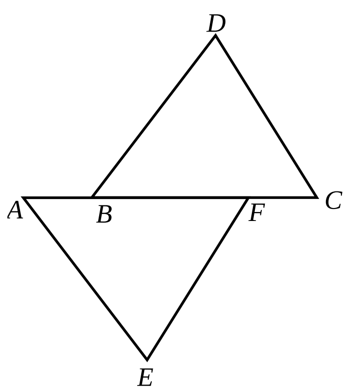
$\therefore$ $\Delta ABC \cong \Delta CDA$

{other results: $\angle 3 = \angle 4$, BC = AD, $\angle B = \angle D$ (c.p.c.t.)

We can not use 'RHS' for this proof.

Note : ASS or SSA criteria for congruency is not valid.

Ex.2 In Fig. it is given that AB = CF, EF = BD and $\angle AFE = \angle DBC$. Prove that $\Delta AFE \cong \Delta CBD$.



Sol. We have, AB = CF

$\Rightarrow$ AB + BF = CF + BF

$\Rightarrow$ AF = CB .... (i)

In Δs AFE and CBD, we have

AF = CB [From (i)]

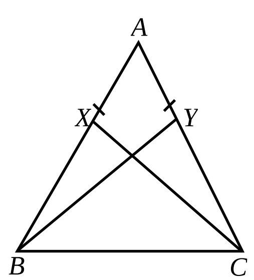
$\angle AFE = \angle DBC$ [Given]

and EF = BD [Given]

So, by SAS criterion of congruence, we have

$\Delta AFE \cong \Delta CBD$

Ex.3 In Fig. X and Y are two points on equal sides AB and AC of a ΔABC such that AX = AY. Prove that XC = YB.



Sol. In Δs AXC and AYB, we have

AX = AY [Given]

$\angle A = \angle A$ [Common angle]

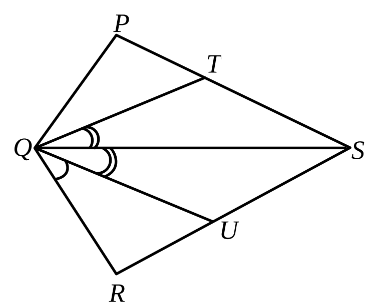
AC = AB [Given]

So, by SAS criterion of congruene

$\Delta AXC \cong \Delta AYB$

$\Rightarrow$ XC = YB (c.p.c.t.)

Ex.4 In Fig. PQRS is a quadrilateral and T and U are respectively points on PS and RS such PQ = RQ, $\angle PQT = \angle RQU$ and $\angle TQS = \angle UQS$. Prove that QT = QU.



Sol. We have,

$\angle PQT = \angle RQU$

and, $\angle TQS = \angle UQS$

$\therefore$ $\angle PQT + \angle TQS = \angle RQU + \angle UQS$

$\Rightarrow$ $\angle PQS = \angle RQS$ .... (i)

Thus, in triangles PQS and RQS, we have

PQ = RQ [Given]

$\angle PQS = \angle RQS$ [From (i)]

and, QS = QS [Common side]

Therefore, by SAS congruence criterion, we have

$\Delta PQS \cong \Delta RQS$

$\Rightarrow$ $\angle QPS = \angle QRS$ (c.p.c.t.)

$\Rightarrow$ $\angle QPT = \angle QRU$ ....(ii)

Now, consider triangles QPT and QRS. In these two triangles, we have

QP = QR [Given]

$\angle PQT = \angle RQU$ [Given]

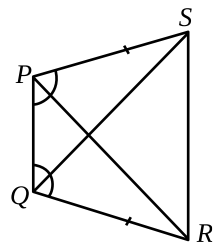
$\angle QPT = \angle QRU$ [From (ii)]

Therefore, by ASA congruence criterion, we get

$\Delta QPT \cong \Delta QRU$

$\Rightarrow$ QT = QU.

Ex.5 In Fig. PS = QR and $\angle SPQ = \angle RQP$.



Prove that PR = QS and $\angle QPR = \angle PQS$.

Sol. In ΔSPQ and ΔRQP, we have

PS = QR [Given]

$\angle SPQ = \angle RQP$ [Given]

PQ = PQ [Common]

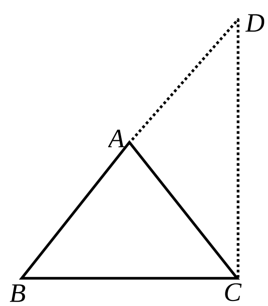
Therefore, by SAS criterion of congruence, we have

$\Delta SPQ \cong \Delta RQP \Rightarrow$ SQ = RP and

$\angle QPR = \angle PQS$

Ex.6 ΔABC is an isosceles triangle with AB = AC. Side BA is produced to D such that AB = AD. Prove that $\angle BCD$ is a right angle.

Sol. Given : A ΔABC such that AB = AC. Side BA is produced to D such that AB = AD.



Construction : Join CD.

To prove : $\angle BCD = 90^\circ$

Proof : In ΔABC, we have AB = AC

$\Rightarrow \angle ACB = \angle ABC$ ...(i) $\left[\begin{array}{c}\because \text{Angles opp. to} \\ \text{equal sides are equal}\end{array}\right]$

Now, AB = AD [Given]

$\therefore$ AD = AC [$\therefore$ AB = AC]

Thus, in ΔADC, we have

AD = AC

$\Rightarrow \angle ACD = \angle ADC$ …(ii)

[$\because$ Angles opp. to equal sides are equal]

Adding (i) and (ii), we get

$\angle ACB + \angle ACD = \angle ABC + \angle ADC$

$\Rightarrow \angle BCD = \angle ABC + \angle BDC$

[$\because \angle ADC = \angle BDC, \angle ABC = \angle DBC$]

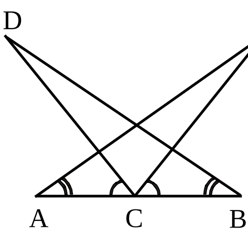
$\Rightarrow \angle BCD + \angle BCD = \angle DBC + \angle BCD + \angle BDC$ $\left[\begin{array}{c}\text{Adding } \angle BCD \\ \text{on both side}\end{array}\right]$

$\Rightarrow 2\angle BCD = 180^\circ$

[$\because$ Sum of the angles of a Δ is 180°]

Hence, $\angle BCD$ is a right angle.

Ex.7 In Fig. AC = BC, $\angle DCA = \angle ECB$ and $\angle DBC = \angle EAC$.



Prove that triangles DBC and EAC are congruent, and hence DC = EC.

Sol. We have,

$\angle DCA = \angle ECB$

$\Rightarrow \angle DCA + \angle ECD = \angle ECB + \angle ECD$

$\Rightarrow \angle ECA = \angle DCB$ .... (i)

Now, in Δs DBC and EAC, we have

$\angle DCB = \angle ECA$ [From (i)]

BC = AC [Given]

and $\angle DBC = \angle EAC$ [Given]

So, by ASA criterion of congruence

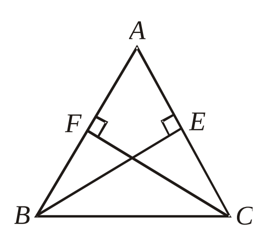
$\Delta DBC \cong \angle EAS$

$\Rightarrow$ DC = EC

(c.p.c.t.)

Ex.8 If the altitudes from two vertices of a triangle to the opposite sides are equal, prove that the triangle is isosceles.

Sol. Given : A ΔABC in which altitudes BE and CF from B and C respectively on AC and AB are equal.



To prove : ΔABC is isoceles i.e. AB = AC

Proof : In Δs ABC and ACF, we have

$\angle AEB = \angle AFC$ [Each equal to 90º]

$\angle BAE = \angle CAF$ [Common angle]

and, $BE = CF$ [Given]

So, by AAS criterion of congurence, we have

$\Delta ABE \cong \Delta ACF$

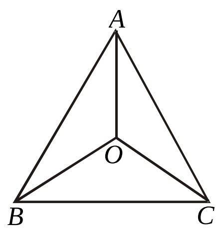
$\Rightarrow AB = AC$ $\left[\because \text{Corresponding parts of congruent triangles are equal}\right]$

Hence, ΔABC is isosceles.

Ex.9 In ΔABC, AB = AC and the bisectors of angles B and C intersect at point O. Prove that BO = CO and the ray AO is the bisector of angle BAC.

Sol. In ΔABC, we have

$AB = AC$



$\Rightarrow \angle B = \angle C$ $\left[\because \text{Angles opposite to equalsides are equal}\right]$

$\Rightarrow \frac{1}{2}\angle B = \angle BC\frac{1}{2}$

$\Rightarrow \angle OBC = \angle OCB$ .... (i)

$\left[\because \text{ OB and OC are bisectors of } \angle\text{s B and C respectively } \therefore \angle OBC = \frac{1}{2}\angle B \ \& \ \angle OCB = \frac{1}{2}\angle C\right]$

$\Rightarrow OB = OC$ ....(ii)

[$\because$ Sides opp. to equal $\angle$s are equal]

Now, in ΔABO and ΔACO, we have

$AB = AC$ [Given]

$\angle OBC = \angle OCB$ [From (i)]

$OB = OC$ [From (ii)]

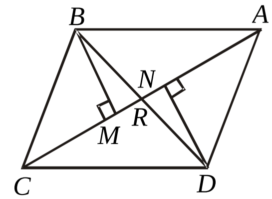
So, by SAS criterion of congruence

$\Delta ABO \cong \Delta ACO$

$\Rightarrow \angle BAO = \angle CAO$ [$\because$ Corresponding parts of congruent triangles are equal]

$\Rightarrow$ AO is the bisector of $\angle BAC$.

Ex.10 In Fig. BM and DN are both perpendiculars to the segments AC and BM = DN.



Prove that AC bisects BD.

Sol. In Δs BMR and DNR, we have

$\angle BMR = \angle DNR$

[Each equal to 90º $\because$ BM $\perp$ AC and DN $\perp$ AC]

$\angle BRM = \angle DRN$ [Vert. opp. angles]

and, $BM = DN$ [Given]

So, by AAS criterion of congruence

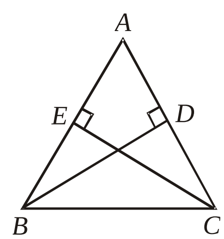
$\Delta BMR \cong \Delta DNR$

$\Rightarrow BR = DR$ $\left[\because \text{Corresponding parts of congruent triangles are equal}\right]$

$\Rightarrow$ R is the mid-point of BD.

Hence, AC bisects BD.

Ex.11 In Fig. BD and CE are two altitudes of a ΔABC such that BD = CE.



Prove that ΔABC is isolceles.

Sol. In ΔABD and ΔACE, we have

$\angle ADB = \angle AEC = 90º$ [Given]

$\angle BAD = \angle CAE$ [Common]

and, $BD = CE$ [Given]

So, by AAS congruence criterion, we have

$\Delta ABD \cong \Delta ACE$

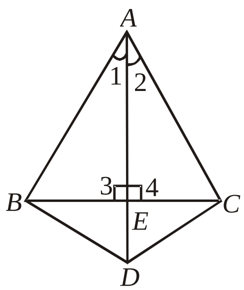
$\Rightarrow AB = AC$ $\left[\because \text{Corresponding parts of congruent triangles are equal}\right]$

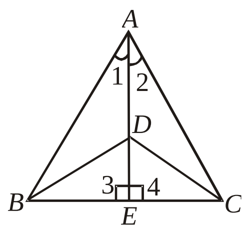
Hence, ΔABC is isosceles.

Ex.12 If two isosceles triangles have a common base, the line joining their vertices bisects them at right angles.

Sol. **Given :** Two isosceles triangles ABC and DBC having the common base BC such that AB = AC and DB = DC.

To prove : AD (or AD produced) bisects BC at right angle.





Proof : In Δs ABD and ACD, we have

AB = AC [Given]

BD = CD [Given]

AD = AD [Common side]

So, by SSS criterion of congruence

$\Delta ABD \cong \Delta ACD$

$\Rightarrow \angle 1 = \angle 2$ .... (i)

$\left[\because \text{Corresponding parts of congruent triangles are equal}\right]$

Now, in Δs ABE and ACE, we have

AB = AC [Given]

$\angle 1 = \angle 2$ [From (i)]

and, AE = AE [Commoni side]

So, by SAS criterion of congruence,

$\Delta ABE \cong \Delta ACE$

$\Rightarrow$ BE = CE $\left[\because \text{Corresponding parts of congruent triangles are equal}\right]$

and, $\angle 3 = \angle 4$

But, $\angle 3 + \angle 4 = 180^\circ$

[$\because$ Sum of the angles of a linear pair is 180º]

$\Rightarrow 2\angle 3 = 180^\circ$ [$\because \angle 3 = \angle 4$]

$\Rightarrow \angle 3 = 90^\circ$

$\therefore \angle 3 = \angle 4 = 90^\circ$

Hence, AD bisects BC at right angles.

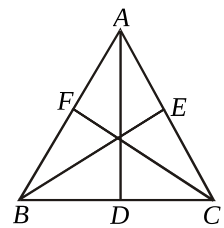
Ex.13 AD, BE and CF, the altitudes of ΔABC are equal. Prove that ΔABC is an equilateral triangle

Sol. In right triangles BCE and BFC, we have

Hyp. BC = Hyp. BC

BE = CF [Given]

So, by RHS criterion of congruence,



$\Delta BCE \cong \Delta BFC$.

$\Rightarrow \angle B = \angle C$ $\left[\because \text{Corresponding parts of congruent triangles are equal}\right]$

$\Rightarrow$ AC = AB .... (i)

[$\because$ Sides opposite to equal angles are equal]

Similarly, $\Delta ABD \cong \Delta ABE$

$\Rightarrow \angle B = \angle A$

[Corresponding parts of congruent triangles are equal]

$\Rightarrow$ AC = BC ....(ii)

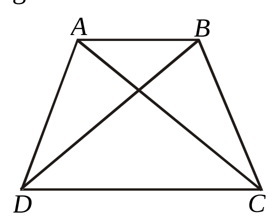
[Sides opposite to equal angles are equal]

From (i) and (ii), we get

AB = BC = AC

Hence, $\Delta\angle ABC$ is an equilateral triangle.

Ex.14 In Fig. AD = BC and BD = CA.



Prove that $\angle ADB = \angle BCA$ and

$\angle DAB = \angle CBA$.

Sol. In triangles ABD and ABC, we have

AD = BC [Given]

BD = CA [Given]

and AB = AB [Common]

So, by SSS congruence criterion, we have

$\Delta ABD \cong \angle CBA \Rightarrow \angle DAB = \angle ABC$

$\left[\because \text{corresponding parts of congruent triangles are equal}\right]$

$\Rightarrow \angle DAB = \angle CBA$

Ex.15 Line-segment AB is parallel to another line-segment CD. O is the mid-point of AD (see figure). Show that (i) $\Delta AOB \cong \Delta DOC$ (ii) O is also the mid point of BC.

Sol. (i) Consider ΔAOB and ΔDOC

$\angle ABO = \angle DCO$

(Alternate angles as AB || CD

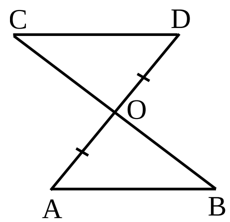
and BC is the transversal)

$\angle AOB = \angle DOC$

(Vertically opposite angles)

OA = OD (Given)

Therefore, $\Delta AOB \cong \Delta DOC$ (AAS rule)

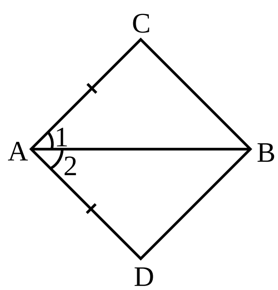


(ii) OB = OC (c.p.c.t.)

So, O is the mid-point of BC.

Ex.16 In quadrilateral ABCD,

AC = AD and AB bisects $\angle A$. Show that $\Delta ABC \cong \Delta ABD$. What can you say about BC and BD ? **[NCERT]**



Sol. In ΔABC & ΔABD

AB = AB (common)

$\angle 1 = \angle 2$ {$\because$ AB is bisector of $\angle A$}

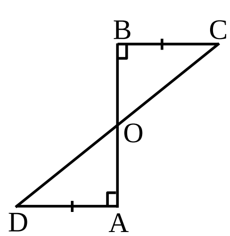
AC = AD (Given)

$\therefore$ By SAS, $\Delta ABC \cong \Delta ABD$ Proved

also BC = BD (c.p.c.t.)

Ex.17 AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB.

[NCERT]



Sol. To show CD bisect AB i.e. AO = OB

$\therefore$ in ΔOAD and ΔOBC

$\angle O = \angle O$ (vertically opposite angles)

$\angle A = \angle B = 90°$ (Given)

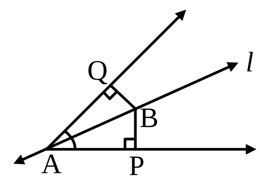
AD = BC (Given)

$\therefore$ By AAS, $\Delta OAD \cong \Delta OBC$

$\therefore$ OA = OB (c.p.c.t.)

$\therefore$ CD, bisects AB. Proved

Ex.18 Line *l* is the bisector of an angle $\angle A$ and B is any point on *l*. BP and BQ are perpendiculars from B to the arms of $\angle A$ (see figure). Show that : **[NCERT]**



(i) $\Delta APB \cong \Delta AQB$

(ii) BP = BQ or B is equidistant from the arms of $\angle A$.

Sol. (i) In ΔAPB and ΔAQB

$\angle P = \angle Q = 90°$ (Given)

$\angle PAB = \angle QAB$ (Given that '*l*' bisect $\angle A$)

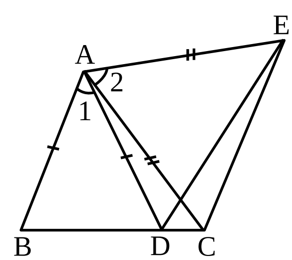
AB = AB (Common)

$\therefore$ By AAS, $\Delta APB \cong \Delta AQB$. Proved

(ii) BP = BQ (c.p.c.t.) Proved.

Ex.19 In given figure, AC = AE, AB = AD and $\angle BAD = \angle EAC$. Show that BC = DE.

[NCERT]



Sol. In ΔABC and ΔADE

AB = AD (Given)

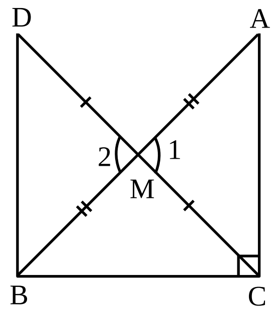
$$\angle BAC = \angle DAE \left\{ \begin{array}{ll} \because \angle 1 = \angle 2 & \text{Given} \\ \angle 1 + \angle DAC = \angle 2 + \angle DAC \end{array} \right\}$$

AC = AE (Given)

$\therefore$ By SAS, $\Delta ABC \cong \Delta ADE$

$\therefore$ BC = DE (c.p.c.t.) Proved.

Ex.20 In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see figure). Show that: **[NCERT]**



(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle

(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2} AB$

Sol. (i) In ΔAMC and ΔBMD

AM = MB (M is mid point of AB)

$\angle 1 = \angle 2$ (vertically opposite angles)

CM = MD (given)

$\therefore$ By SAS, $\Delta AMC \cong \Delta MBD$ Proved.

(ii) $\angle ACM = \angle MDB$ (c.p.c.t. of (i))

These are alternate angles

$\therefore$ DB || AC

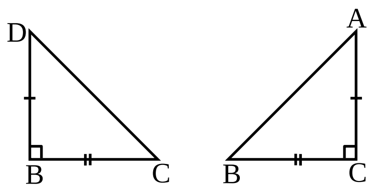
So $\angle DBC + \angle ACB = 180°$

(Cointerior angles)

$\Rightarrow \angle DBC + 90° = 180°$

$\Rightarrow \angle DBC = 90°$ Proved.

(iii)



In ΔDBC & ΔACB

BC = BC (common)

$\angle DBC = \angle ACB = 90°$

DB = AC (c.p.c.t. of part (i))

$\therefore$ By SAS, $\Delta DBC \cong \Delta ACB$. Proved

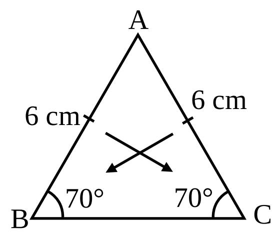
(iv) DC = AB (c.p.c.t. of part (iii))

But $CM = \frac{1}{2} DC$ (given)

$\therefore CM = \frac{1}{2} AB$ Proved.

➤ ISOSCELES TRIANGLE

A triangle in which two sides are equal & opposite angles of these two lines are also equal.



AB = AC = 6 cm, $\angle B = \angle C = 70°$

Ex.21 Find $\angle BAC$ of an isosceles triangle in which AB = AC and $\angle B = \frac{1}{3}$ of right angle.

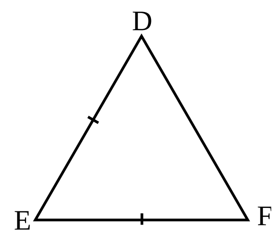
Sol. $\angle B = \angle C = \frac{1}{3}(90) = 30°$

$\therefore \angle A + \angle B + \angle C = 180°$ (ℓ.p.)

$\angle A + 30° + 30° = 180° \Rightarrow \angle A = 120°$.

Ex.22 In isosceles triangle DEF, DE = EF and $\angle E = 70°$ then find other two angles.

Sol.



Let $\angle D = \angle F = x$

$\therefore \angle D + \angle E + \angle F = 180°$

(angle sum property)

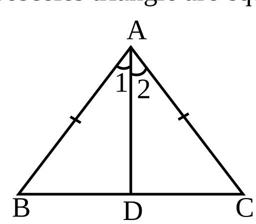
$\Rightarrow x + 70° + x = 180°$

$\Rightarrow 2x = 110°$

$\Rightarrow x = 55°$

$\Rightarrow \angle D = \angle F = 55°$.

Theorem (2) : Angles opposite to equal sides of an isosceles triangle are equal.



Given : In ΔABC, AB = AC

To prove : $\angle B = \angle C$

Construction : Draw AD, bisector of $\angle A$

$\therefore \angle 1 = \angle 2$

Proof : In ΔADB & ΔADC

AD = AD (Common)

$\angle 1 = \angle 2$ (by construction)

AB = AC

By SAS, $\Delta ADB \cong \Delta ADC$

$\therefore \angle B = \angle C$ (c.p.c.t.) Proved.

Note : Other result : $\angle ADB = \angle ADC$ (c.p.c.t.)

But $\angle ADB + \angle ADC = 180°$ (linear pair)

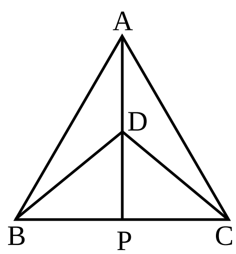
$\therefore \angle ADB = \angle ADC = 90° \Rightarrow AD \perp BC$

and BD = DC (c.p.c.t.) $\Rightarrow$ AD is median

$\therefore$ we can say AD is perpendicular bisector of BC or we can say in isosceles Δ, median is angle bisector and perpendicular to base also.

Ex.23 ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see fig.). If AD is extended to intersect BC at P. Show that

[NCERT]



(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$

(iii) AP bisects $\angle A$ as well as $\angle D$

(iv) AP is the perpendicular bisector of BC.

Sol. (i) In ΔABD & ΔACD

AB = AC ($\because$ ΔABC is isosceles Δ)

AD = AD (Common)

BD = DC (ΔDBC is isosceles Δ)

$\therefore$ By SSS, $\Delta ABD \cong \Delta ACD$ Proved.

(ii) In ΔABP & ΔACP

AB = AC ($\because$ ΔABC is isosceles Δ)

$\angle ABP = \angle ACP$ {$\because$ ΔABC is isosceles Δ}

AP = AP (common)

$\therefore$ By SAS, $\Delta ABP \cong \Delta ACP$ Proved.

(iii) $\because \angle BAP = \angle CAP$ (c.p.c.t. of part (ii))

$\therefore \angle A$ is bisected by AP

and $\angle ADB = \angle ADC$ (c.p.c.t. of part (ii))

$\therefore$ CD is bisected by AP.

(iv) $\angle APB = \angle APC$ (c.p.c.t. of part (ii))

but $\angle APB + \angle APC = 180°$ (linear pair)

$\therefore \angle APB + \angle APB = 180°$

$2\angle APB = 180°$

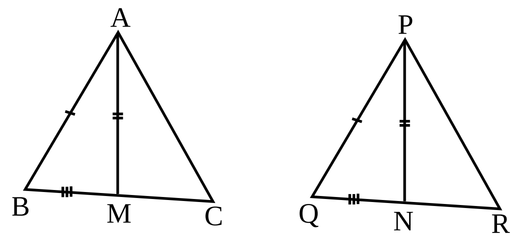
$\angle APB = 90° = \angle APC$

also PB = PC (c.p.c.t. of part (ii))

$\therefore$ AP is perpendicular bisector of BC.

Proved.

Ex.24 Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (see figure). Show that:



(i) $\Delta ABM \cong \Delta PQN$ (ii) $\Delta ABC \cong \Delta PQR$

Sol. (i) In ΔABM & ΔPQN

AB = PQ (given)

AM = PN (given)

BM = QN ($\because$ BC = QR

$\therefore \frac{BC}{2} = \frac{QR}{2}$)

$\therefore$ By SSS, $\Delta ABM \cong \Delta PQN$ Proved.

(ii) In ΔABC & ΔPQR

AB = PQ (given)

$\angle B = \angle Q$ (c.p.c.t. of part (i))

BC = QR (given)

$\therefore$ By SAS, $\Delta ABC \cong \Delta PQR$ Proved.

➢ SOME MORE RESULTS BASED ON CONGRUENT TRIANGLES

(1) If two sides of a triangle are unequal, then the longer side has the greater angle opposite to it.

(2) In a triangle, the greater angle has the longer side opposite to it.

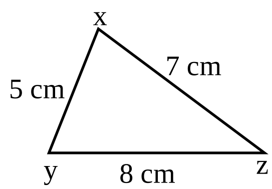
(3) Of all the line segments that can be drawn to a given line, from a point not lying on it, the perpendicular line segment is the shortest.

(4) The sum of any two sides of a triangle is greater than its third side.

(5) The difference between any two sides of a triangle is less than its third side.

(6) Exterior angle is greater than one opposite interior angle.

Ex.25 Find the relation between angles in figure.

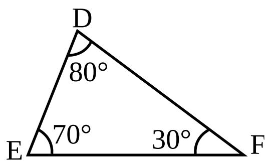


Sol. $\because$ yz > xz > xy

$\therefore$ $\angle x > \angle y > \angle z$.

($\because$ Angle opposite to longer side is greater)

Ex.26 Find the relation between the sides of triangle in figure .

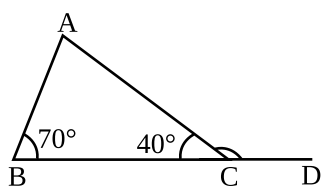


Sol. $\because$ $\angle D > \angle E > \angle F$

$\therefore$ EF > DF > DE

{$\because$ side opposite to greater angle is longer}

Ex.27 Find $\angle ACD$ then what is the relation between (i) $\angle ACD$, $\angle ABC$ (ii) $\angle ACD$ & $\angle A$



Sol. $\angle ACD + 40° = 180°$ (linear pair)

$\angle ACD = 140°$ **Ans.**

also $\angle A + \angle B = \angle ACD$

(exterior angle = sum of opp. interior angles)

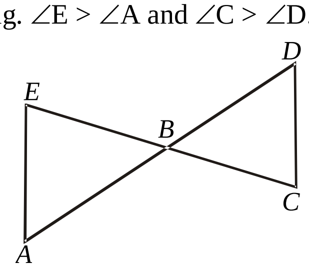
$\Rightarrow \angle A + 70° = 140° \Rightarrow \angle A = 140° - 70°$

$\Rightarrow \angle A = 70°$

Now $\angle ACD > \angle B$ **Ans.**

$\angle ACD > \angle A$ **Ans.**

Ex.28 In Fig. $\angle E > \angle A$ and $\angle C > \angle D$.



Prove that AD > EC.

Sol. In ΔABE, it is given that

$\angle E > \angle A$

$\Rightarrow$ AB > BE .... (i)

In ΔBCD, it is given that

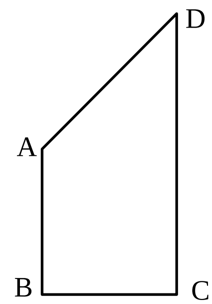
$\angle C > \angle D$

$\Rightarrow$ BD > BC ....(ii)

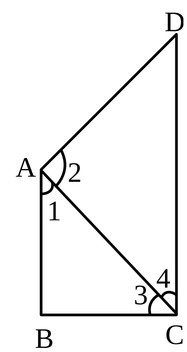
Adding (i) and (ii), we get

AB + BD > BE + BC $\Rightarrow$ AD > EC

Ex.29 AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see figure). Show that $\angle A > \angle C$. **[NCERT]**



Sol. Draw diagonal AC.



In ΔABC, AB < BC {$\because$ AB is smallest}

$\Rightarrow \angle 3 < \angle 1$ ……(1)

{angle opp. to longer side is larger}

Also in ΔADC

AD < CD $\because$ CD is longest

$\Rightarrow \angle 4 < \angle 2$ …..(2)

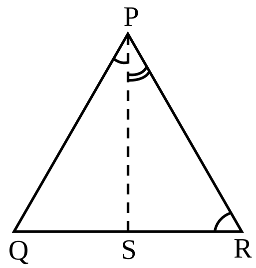
adding equation (1) & (2)

$\angle 3 + \angle 4 < \angle 1 + \angle 2$

$\angle C < \angle A$

or $\angle A > \angle C$ Proved.

Ex.30 In given figure, PR > PQ and PS bisects $\angle QPR$. Prove that $\angle PSR > \angle PSQ$. **[NCERT]**

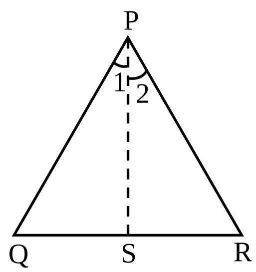


Sol. In ΔPQR, $PR > PQ$

$\Rightarrow \angle Q > \angle R$ ……(1)

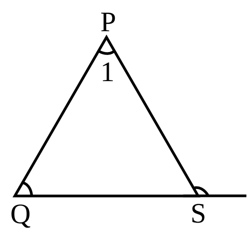
{angle opposite to longer side is greater}

and $\angle 1 = \angle 2$ ($\because$ PS is $\angle$ bisector) …..(2)



Now for ΔPQS, $\angle PSR = \angle Q + \angle 1$ …..(3)

{exterior angle = sum of opposite interior angle}



& for ΔPSR, $\angle PSQ = \angle R + \angle 2$ …..(4)

By equation (1), (2), (3), (4), $\angle PSR > \angle PSQ$

Proved.

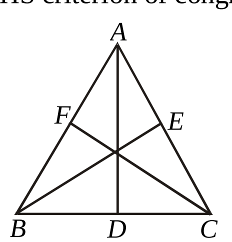
Ex.31 AD, BE and CF, the altitudes of ΔABC are equal. Prove that ΔABC is an equilateral triangle

Sol. In right triangles BCE and BFC, we have

Hyp. BC = Hyp. BC

BE = CF [Given]

So, by RHS criterion of congruence,



$\Delta BCE \cong \Delta BFC$.

$\Rightarrow \angle B = \angle C$ $\left[\begin{array}{l}\because \text{Corresponding parts of} \\ \text{congruent triangles are equal}\end{array}\right]$

$\Rightarrow$ AC = AB .... (i)

[$\because$ Sides opposite to equal angles are equal]

Similarly, $\Delta ABD \cong \Delta ABE$

$\Rightarrow \angle B = \angle A$

[Corresponding parts of congruent triangles are equal]

$\Rightarrow$ AC = BC ....(ii)

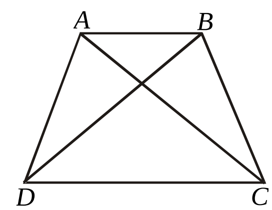
[Sides opposite to equal angles are equal]

From (i) and (ii), we get

AB = BC = AC

Hence, ΔABC is an equilateral triangle.

Ex.32 In Fig. AD = BC and BD = CA.



Prove that $\angle ADB = \angle BCA$ and

$\angle DAB = \angle CBA$.

Sol. In triangles ABD and ABC, we have

AD = BC [Given]

BD = CA [Given]

and AB = AB [Common]

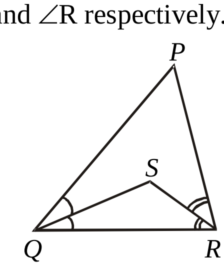
So, by SSS congruence criterion, we have

$\Delta ABD \cong \angle CBA \Rightarrow \angle DAB = \angle ABC$

$\left[\begin{array}{l}\because \text{corresponding parts of} \\ \text{congruent triangles are equal}\end{array}\right]$

$\Rightarrow \angle DAB = \angle CBA$

Ex.33 In Fig. PQ > PR. QS and RS are the bisectors of $\angle Q$ and $\angle R$ respectively.



Prove that SQ > SR.

Sol. In Δ PQR, we have

PQ > PR [Given]

$\Rightarrow \angle PRQ > \angle PQR$ $\left[\begin{array}{l}\text{Angle opp. to larger side} \\ \text{of a triangle is greater}\end{array}\right]$

$\Rightarrow \ \frac{1}{2}\angle PRQ > \frac{1}{2}\angle PQR$

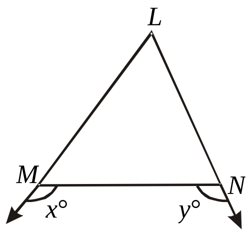
$\Rightarrow \ \angle SRQ > \angle SQR$

$\begin{bmatrix} \because \text{RS and QS are bisectors of} \\ \angle PRQ \text{ are } \angle PQR \text{ respectively} \end{bmatrix}$

$\Rightarrow$ SQ > SR

[$\because$ Side opp. to greater angle is larger]

Ex.34 In Fig. **[NCERT]**



if x > y, show that $\angle M > \angle N$.

Sol. We have,

$\angle LMN + x^o = 180^o$ .... (i)

[Angles of a linear pair]

$\Rightarrow \ \angle LNM + y^o = 180^o$ ....(ii)

[Angles of a linear pair]

$\therefore \ \angle LMN + x^o = \angle LNM + y^o$

But x > y. Therefore,

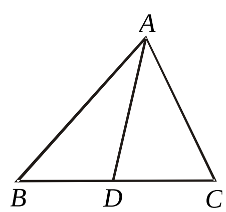
$\angle LMN < \angle LNM$

$\Rightarrow \ \angle LNM > \angle LMN$

$\Rightarrow$ LM > LN

[$\because$ Side opp. to greater angle is larger]

Ex.35 In Fig. AB > AC. Show that AB > AD.



Sol. In ΔABC, we have

AB > AC [Given]

$\Rightarrow \ \angle ACB > \angle ABC$ .... (i)

[$\because$ Angle opp. to larger side is greater]

Now, in ΔACD, CD is produced to B, forming an ext $\angle ADB$.

$\therefore \ \angle ADB > \angle ACD$

$\begin{bmatrix} \because \text{Exterior angle of } \Delta \text{ is greater} \\ \text{than each of interior opp. angle} \end{bmatrix}$

$\Rightarrow \ \angle ADB > \angle ACB$ ... (ii)

[$\therefore \ \angle ACD = \angle ACB$]

From (i) and (ii), we get

$\angle ADB > \angle ABC$

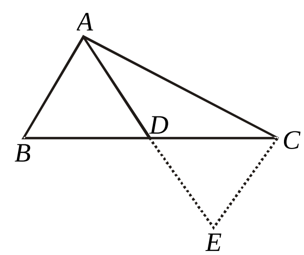
$\Rightarrow \ \angle ADB > \angle ABD$ [$\because \angle ABC = \angle ABD$]

$\Rightarrow$ AB > AD

[$\because$ Side opp. to greater angle is larger]

Ex.36 Prove that any two sides of a triangle are together greater than twice the median drawn to the third side.

Sol. **Given :** A ΔABC in which AD is a median.



To prove : AB + AC > 2 AD

Construction : Produce AD to E such that AD = DE. Join EC.

Proof : In Δs ADB and EDC, we have

AD = DE [By construction]

BD = DC [$\because$ D is the mid point of BC]

and, $\angle ADB = \angle EDC$ [Vertically opp. angles]

So, by SAS criterion of congruence

$\Delta ADB \cong \Delta EDC$

$\Rightarrow$ AB = EC $\begin{bmatrix} \text{Corresponding parts of} \\ \text{congruent triangles are equal} \end{bmatrix}$

Now in ΔAEC, we have

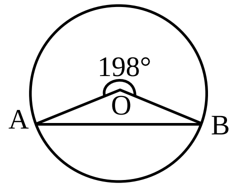
AC + EC > AE [$\because$ Sum of any two sides of a Δ is greater than the third]

$\Rightarrow$ AC + AB > 2 AD

$[\because AD = DE \therefore AE = AD + DE = 2AD \text{ and } EC = AB]$

Ex.40 Find $\angle OBA$ in given figure

Sol.



$\because \angle AOB + 198° = 360°$

$\angle AOB = 360° – 198° = 162°$

and OA = OB = radius of circle

$\angle A = \angle B = x$ (let)

$\therefore x + x + 162° = 180°$ (a.s.p.)

$2x + 18°$

$x = 9°$

$\therefore \angle OBA = 9°$.

IMPORTANT POINTS TO BE REMEMBERED

1. A palne figure bounded by three lines in a plane is called a triangle.

2. A triangle, no two of whose sides are equal is called a scalene triangle.

3. Atriangle whose two sides are equal is called an isosceles triangle.

4. A triangle whose sides are equal is also called an equilateral triangle.

5. A triangle with one angle a right angle is called a right angled triangle.

6. The sum of the three angles of a triangle is 180º.

7. If a side of a triangles is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

8. If two triangles ABC and DEF are congruent under the correspondence $A \leftrightarrow D$, $B \leftrightarrow E$ and $C \leftrightarrow F$, then we write $\Delta ABC \cong \Delta DEF$ or $\Delta ABC \leftrightarrow \Delta DEF$.

9. Two triangles are congruent if two sides and the included angle of one are equal to the corresponding sides and the included angle of the other triangle (SAS congruence criterion).

10. Two triangles are congruent if two angles and the included side of one tringle are equal to the corresponding two angles and the included side of the other triangle (ASA congruence criterion).

11. If any two angles and non-included side of one triangle are equal to the corresponding angles and side of another triangle, then the triangles are congruent (AAS congruence criterion).

12. If three sides of one triangle are equal to three of the other triangle, then the two triangles are congruent (SSS congruence criterion).

13. If in two right triangles, hypotenuse and one side of a triangle are equal to the hypotenuse and one side of other triangle, then the two triangles are congruent (RHS congruence criterion).

14. Angles opposite to equal sides of a triangle are equal.

15. If the altitude from one vertex of a triangle bisects the opposite sides, then the triangle is isosceles.

16. In an isosceles triangle altitude from the vertex bisects the base.

17. If the bisector of the vertical angle of a triangle bisects the opposite side, then the triangle is isosceles.

18. If the altitudes of a triangles are equal, then it is equilateral.

19. In a triangle, side opposite to the larger angle is longer.

20. Sum of any two sides of a triangle is greater than the third side.